A High-Precision Positioning System Using Magnetic Levitation*

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INTRODUCTION

High accuracy positioning stages with multiple degrees-of-freedom are ubiquitous in industrial manufacturing. Examples include probing and inspection systems, disk drive head assembly systems, and photolithographic positioning stages (microsteppers) in semiconductor manufacturing. Microsteppers are typically made of a lower-stage that actuates large high-speed movements and an upper-stage that delivers high-precision movements using piezoelectric actuators [1]. One of the main drawbacks of mechanical positioning actuators in semiconductor manufacturing is that mechanical contacts introduce impurities limiting the accuracy of the photolithographic process, thus reducing production throughput. Furthermore, mechanical positioning devices require costly maintenance due to the wear of their components. These problems become critical as the pace of technology causes the overall dimensions of semiconductors to further decrease. For these reasons, there is an increasing interest, in industry, to replace mechanical microsteppers by contactless positioning devices.

While high-precision contactless positioning devices have obvious application in semiconductor manufacturing, it is also clear that they could be used in any area where speed and

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precision are required to control position with multiple degrees of freedom, and where the aforementioned limitations of mechanical positioning are a serious concern.

In [2], Kim and Trumper proposed a contactless microstepper which employs single sided air cored permanent magnet linear synchronous motors (PMLSM) to actuate six degrees-of-freedom. Individually, PMLSMs produce both a normal and translational force with appropriate control. When several are combined in appropriate fashion, multiple degrees of freedom can be achieved.

In this paper we continue the research initiated in [3], where an idealized positioning device employing three flat single sided iron cored PMLSMs to position a platen with three DOF was developed. This device was designed to work over a large range of operation and to employ standard PMLSMs commonly found on the market. While the idealized models and control designs from [3] were promising, it remained to be seen whether the results could be translated into a working physical realization. This paper presents the experimental verification of the theory developed in [3] using a simplified experimental apparatus which employs one PMLSM actuating two degrees-of-freedom, shown in Figure 1. This apparatus represents the simplest experimental setup needed to test the accuracy of the model and the adequacy of the nonlinear control design, and is therefore to be considered a first-generation proof of concept. The apparatus we present was developed in collaboration with Quanser Inc.

We show, by means of the parameter identification technique developed by Pan and Başar in [4], that the mathematical model in [3] is accurate over a wide range of operation (100mm ×10mm). When our nonlinear controller is employed to control the system, the experimental results show that set-point stabilization and sinusoidal tracking can be achieved with performance that meets our goals (set-point tracking error below 0.1mm with settling time below 3s and sinusoidal tracking error close to .1mm). We also show that the nonlinear controller performs significantly better than some linear counterparts.

We begin our discussion with a description of the initial problem formulation. This is followed by the implementation details of the first-generation magnetic levitation hardware utilized during experimentation, along with a brief summary of the model derivation. We proceed to demonstrate how a parameter identification technique developed in [4] was used
to estimate the parameters of the model in [3]. This model was then verified by comparing the forces and currents predicted by the model to the actual forces and currents produced by our apparatus. Once the final model and validation procedure have been presented, we summarize the design, implementation, and testing process of a nonlinear controller based on feedback linearization and output regulation. Detailed experimental results are included.

**MOTIVATION AND GOALS**

As mentioned in the introduction, our main motivation in implementing the apparatus depicted in Figure 1 is to prove the modeling and control concepts as laid out in [3] using the simplest possible experimental setup, one linear motor actuating two degrees-of-freedom.

Our first goal is to determine how accurately the system model could predict the physical behavior of the magnetic levitation apparatus. Using a parameter identification technique, we develop a simple experimental procedure which determines the operating range over which the mathematical model developed in [3] can be deemed accurate.

The second goal is the development of a nonlinear controller. For this first-generation experiment, it is desired that the nonlinear controller be capable of both set-point stabilization and sinusoidal tracking over the operating range of the apparatus. We seek to achieve a tracking error below 0.1mm for set-point stabilization, and a reasonable settling time (while no specific settling time is mandated, anything beyond three seconds is considered unreasonable). For sinusoidal tracking, it is desired that the nonlinear controller achieve a tracking error as close to 0.1mm as possible. Beyond these specifications, no other requirements are placed upon the system performance.

A third goal is to compare the performance of our nonlinear controller to both a linearization-based output regulation controller and a PID controller. The latter comparison is particularly interesting because magnetic levitation systems are often controlled using classical PID controllers.

As future generations of magnetic levitation apparatus are constructed, a set of more rigid performance specifications will eventually be developed. For instance, while the present specification for a steady-state accuracy of 0.1mm is a good start, this still does not approach the accuracy needed in semiconductor manufacturing (which is approaching 0.1µm).
DESCRIPTION OF 2-DOF HARDWARE

As mentioned earlier, the setup in Figure 1, built by Quanser Inc., was constructed from a single iron-cored permanent magnet linear synchronous motor. The physical characteristics of the single motor allow for a horizontal range of motion of approximately ±50mm and a vertical range of motion of approximately 10mm. Detailed hardware specifications are summarized in Table 1. The individual components of the system are described in what follows.

PMLSM Mover and Stator

The stator of the PMLSM, which is fixed in place to a heavy aluminum frame, is longitudinally laminated and transversally slotted in order to accommodate a single layer of 3-phase winding. The mover, which is attached to a movable platform, is composed of a set of four type N35 permanent magnets (PM) attached to a ferromagnetic backing.

Linear Guides, Stoppers and Sensors

The mover of the PMLSM is mounted on a movable platform which constrains the motion to lie on a vertical plane. The platform slides on two horizontal linear guides, shown in Figure 2, which in turn are supported by a base which slides vertically on four linear guides, shown in Figure 3.

Using linear guides in this first-generation experiment has several advantages. First, they allow us to focus on the two degrees-of-freedom we want to control by preventing the platform from pitching, rolling, and translating in an undesired direction. Further, linear guides make it easy to include a set of vertical and horizontal stoppers useful to adjust the operating range of the platform and even to fix either the horizontal or vertical motion of the system. Finally, linear guides allow for easy incorporation of sensors. We employed two linear optical encoders to measure the horizontal and vertical position of the platform. The resulting sensing accuracy is 10µm.

While using linear guides guarantees contactless power transfer, they introduce friction in the system. Linear guides will be eliminated, in future generations of the experiment, by
employing multiple linear motors to control five degrees of freedom.

**Power Delivery**

The 3-phase AC current required to actuate the stator coils is provided by a set of three linear current amplifier modules (LCAM) supplied by Quanser Inc. Current commands are sent to the LCAM’s through an interface from a PC. Each LCAM is capable of supplying 7A continuous and 9A peak. The power delivery is also factory tuned in order to account for the inductance of the stator coils that are used in the system.

**Computer Interface and Real-Time Control Environment**

The magnetic levitation apparatus is connected to a standard PC running Windows XP using a Quanser Multi-Q PCI I/O Board. The Quanser board provides all of the necessary analog and digital I/O, and is operated at a sampling frequency of 2kHz.

Control of the magnetic levitation hardware through the interface is implemented using the Quanser WINCON real-time environment. The software is fully integrated into MATLAB and allows construction of controllers using Simulink diagrams.

**MODEL OF 2-DOF SYSTEM**

The following is a brief summary of the model derivation applied specifically to the 2-DOF system. The details of the modelling are found in [3].

Consider the inertial frame of the single PMLSM that forms the basis of the 2-DOF system, which is shown in Figure 4. Let $L_A$ be the depth of each PM along the $z$ axis, $h_m$ be the height of the magnets, $p_m$ the number of PM’s, $g$ the air-gap length, $t_1$ the slot pitch, $b_0$ the slot aperture, $\tau$ the PM pole pitch, $\tau_p$ the PM pole arc, $\mu_{rec}$ the relative PM recoil permeability, and $\sigma_m$ the surface magnetic charge. We account for the effects of the stator slots by replacing the air-gap $g$ with the effective air-gap $g_e = gK_c(g)$, where $K_c(\cdot)$ denotes
Carter’s coefficient and is given by

\[ K_c = \frac{t_1}{t_1 - g\gamma_1}, \quad \gamma_1 = \frac{4}{\pi} \left[ \frac{b_0}{2g} \arctan \left( \frac{b_0}{2g} \right) - \ln \sqrt{1 + \left( \frac{b_0}{2g} \right)^2} \right]. \]

In addition, let \( I_a, I_b, \) and \( I_c \) be the phasors of the phase currents and \( I_a, I_b, \) and \( I_c \) their magnitudes. Define \( W \) to be the number of turns of wire on each phase, \( p \) the number of pole pairs in the stator, \( w_c \) the coil pitch, and \( k_{w1} \) the winding factor.

We define the horizontal motion to be along the \( x \)-axis while vertical motion is fixed to the \( y \)-axis. Defining \( G \) as the gravitational acceleration constant and \( M_h \) and \( M_v \) as the horizontal and vertical masses of the platform to be levitated, which are different due to the design of the apparatus, one obtains the following model of the 2-DOF system:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= G - L_4(x_1)[u_1^2 + u_2^2] - L_3(x_1)u_2 - L_2(x_1) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -L_1(x_1)u_1.
\end{align*}
\]

where \( x = [g, \dot{g}, d, \ddot{d}]^T \), \( u = [i_q, i_d]^T \), and

\[
\begin{align*}
L_1(x_1) &= \frac{K_1(x_1)}{M_h} \\
L_i(x_1) &= \frac{K_i(x_1)}{M_v}, \quad i = 2, \ldots, 4 \\
K_1(x_1) &= \frac{12\sqrt{2}Wk_{w1}p_{m}L_{A}\sigma_{m}\mu_{0}\tilde{\lambda}(x_1)\sinh(\frac{\pi}{\tau}h_{m})\sin(\frac{\pi\tau}{2\tau})}{\pi pK_c(x_1)\sinh(\frac{\pi}{\tau}(h_{m} + x_1))} \\
K_2(x_1) &= \frac{\tilde{\lambda}(x_1)L_{A}p_{m}\tau B_{pmy1}(x_1)^2}{4\mu_{0}} \\
K_3(x_1) &= -\frac{\tilde{\lambda}(x_1)3\sqrt{2}L_{A}p_{m}Wk_{w1}B_{pmy1}(x_1)\coth(\frac{\pi}{\tau}(h_{m} + x_1))}{p^2 K_c(x_1)} \\
K_4(x_1) &= \frac{18L_{A}p_{m}W^2 k_{w1}^2 \mu_{0} \coth^2(\frac{\pi}{\tau}(h_{m} + x_1))}{\tau p^2 K_c(x_1)^2} \\
\tilde{\lambda}(x_1) &= 1 - \frac{b_0^2}{4t_1(x_1 + \frac{h_{m}}{2} + \frac{b_{m}}{\mu_{rec}})}.
\end{align*}
\]
The function \( B_{pmy1}(x_1) \) represents the magnetic field produced by the PM’s and is approximated using a 12th degree polynomial. Furthermore, \( i_d \) and \( i_q \) represent the direct and quadrature current inputs to the PMLSM. They relate to the 3-phase currents as follows:

\[
\begin{bmatrix}
i_d \\
i_q
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
\cos(\frac{\pi}{3}x_3) & \cos(\frac{2\pi}{3}x_3 - \frac{2\pi}{3}) & \cos(\frac{2\pi}{3}x_3 + \frac{2\pi}{3}) \\
-\sin(\frac{\pi}{3}x_3) & -\sin(\frac{2\pi}{3}x_3 - \frac{2\pi}{3}) & -\sin(\frac{2\pi}{3}x_3 + \frac{2\pi}{3})
\end{bmatrix}
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix}
\]

(2)

The above model does not take into account friction, cogging forces, end effects, and any other physical uncertainty that may be present. It is necessary to verify to what extent such unmodelled effects can be neglected within a reasonable range of operation. This is done in the sections that follow.

As part of this verification of the model, a parameter identification technique will be applied to estimate the constant unknown parameters within the model in (1). In preparation for the application of this technique, the model in (1) is rewritten below with the unknown (or not perfectly known) parameters lumped together into four unknown constants \( C_1, \ldots, C_4 \) as follows

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= G - C_4 \frac{\bar{\lambda}(x_1) \coth^2(\frac{x}{\tau}(h_m + x_1))}{K_c(x_1)} [u_1^2 + u_2^2] \\
&\quad + C_3 \frac{\bar{\lambda}(x_1) B_{pmy1}(x_1) \coth(\frac{x}{\tau}(h_m + x_1))}{K_c(x_1)} u_2 \\
&\quad - C_2 \bar{\lambda}(x_1) B_{pmy1}(x_1)^2 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -C_1 \frac{\bar{\lambda}(x_1)}{K_c(x_1) \sinh(\frac{x}{\tau}(h_m + x_1))} u_1
\end{align*}
\]

MODEL VERIFICATION PROCEDURE

We begin this section with a summary of the parameter identification procedure by Pan and Başar in [4], which we use to estimate the parameters \( C_1, \ldots, C_4 \), together with its implementation. We then employ this procedure to verify the accuracy of the model in (3) and estimate the intensity of the cogging force.
Parameter Estimation Technique

The results from [4] present a series of techniques to estimate constant parameters that enter linearly into nonlinear systems. The methods are based on minimization of a cost-to-come function and formulated in terms of a minimax design problem. We chose to use the reduced-order noise-perturbed full-state information (NPFSI) estimator from [4]. This is reviewed next.

Consider the nonlinear system

\[ \dot{x} = A(x, u)\theta + b(x, u) + \omega(t), \quad x(0) = x_0, \]

(4)

where \( \theta \) is a vector of constant unknown parameters, all functions are smooth, and \( \omega(t) \) is an \( \mathcal{L}_\infty \) disturbance. It is assumed that a noisy measurement of the state \( x \) is available,

\[ y = x + \varepsilon \nu(t), \]

where \( \nu(t) \) is the measurement disturbance and \( \varepsilon \) is a known scalar. The value of \( \varepsilon \) reflects the confidence level in the state measurement. The simplified approximation to the \( \mathcal{H}_\infty \) NPFSI estimator, referred to as the reduced-order NPFSI estimator, was given in [4] as follows:

\[ \dot{\hat{\theta}} = \varepsilon^{-1}\Sigma^{-1}\gamma A(y, u)^T(y - \hat{x}), \quad \dot{\hat{\theta}}(0) = \bar{\theta}_0 \]

(5)

where

\[ \dot{\hat{x}} = A(y, u)\hat{\theta} + b(y, u) + \varepsilon^{-1}(y - \hat{x}), \quad \hat{x}(0) = \bar{x}_0 \]

\[ \dot{\Sigma} = A(y, u)^T A(y, u) - \gamma^{-2}Q(y, u), \quad \Sigma(0) = Q_0. \]

(6)

In the above, \( \gamma \) represents an attenuation factor that can be tuned by the user to improve estimator convergence. It is proven in [4] that as long as the system under consideration satisfies a suitable persistency of excitation condition and \( \varepsilon \) is sufficiently small, the state of (5) converges to the true system parameters.

We next describe the implementation of the estimator (6). Although the NPFSI technique does not require state derivative information, it does depend on measurement of the full state vector. While the position states \( x_1 \) and \( x_3 \) are measured directly using optical encoders, the
velocity states $x_2$ and $x_4$ are not measured. We thus use high-gain observers to estimate $x_2$ and $x_4$. The estimation errors are accounted for, in the parameter identification procedure, by the assumption that a noisy measurement of the states is available. To guarantee the necessary persistency of excitation required by the technique, we employ two PID regulators for the control inputs $i_d$ and $i_q$ to independently make the horizontal and vertical dynamics track a suitable reference signal made of a summation of sinusoids at various frequencies. A block diagram of the PID controlled system is provided in Figure 5. Because the horizontal and vertical dynamics of the system are coupled (see (1)), PID control does not yield good tracking performance. However, since at this stage we only focus on the generation of persistently exciting reference signals for parameter estimation, tracking accuracy is not a concern.

With the above in place, the only remaining issue is the choice of the NPFSI parameters $\gamma$ and $\varepsilon$. It was found through successive experimentation that choosing $\gamma \simeq 1.1$ and $\varepsilon \simeq 0.01$ produced good parameter convergence.

The estimator structure in (6) is used, in what follows, in three different ways. First, to verify the horizontal dynamics. Second, to estimate $C_2$, $C_3$, and $C_4$ and verify the vertical dynamics. Lastly, to estimate $C_1, \ldots, C_4$ and simultaneously verify horizontal and vertical dynamics.

**Verification of Horizontal Dynamics**

If the air-gap of the magnetic levitation system is fixed to a constant value $\bar{x}_1$, then the horizontal dynamics from (1) can be isolated:

\[
\dot{x}_3 = x_4 \\
\dot{x}_4 = -L_1(\bar{x}_1)u_1
\]  

(7)

Since at a fixed air-gap $\bar{x}_1$ the $L_1$ term is constant, the horizontal position $x_3$ can be solved easily:

\[
x_3(t) = -\frac{1}{2}L_1(\bar{x}_1)u_1t^2 + x_3(0)
\]  

(8)

Note that an initial horizontal velocity of zero is assumed. Equation (8) tells us that if a
constant \( u_1 \) is applied to the system when the air-gap is fixed, the horizontal position of the mover exhibits a parabolic response. By recording the horizontal position information subject to these conditions, it should therefore be possible to curve-fit a parabola to the data points and obtain an estimate of \( L_1(\bar{x}_1) \) at the air-gap in question (and therefore an estimate of the horizontal acceleration of the mover). Such estimate is compared to the value of \( L_1(\bar{x}_1) \) obtained through the application of the NPFSI estimator to (7). This idea is summarized below.

- The motion of the system is constrained (by hardware) to lie on the horizontal axis at a fixed air-gap.
- Position data corresponding to different air-gap values \( \{\bar{x}_1^1, \ldots, \bar{x}_1^k\} \) are collected.
- A set of parabolas is fitted to horizontal position data at various air-gaps \( \{\bar{x}_1^1, \ldots, \bar{x}_1^k\} \) in order to obtain an estimate of \( L_1(\bar{x}_i^1), i = 1, \ldots k \) by means of (8).
- For each air-gap \( \bar{x}_1^i \), the NPFSI estimator (6) is applied to (7) to estimate \( L_1(\bar{x}_1^i) \) and the results produced by the two methods are compared.

Figure 6 shows a few examples of the data points obtained and the parabolas that were fitted. Note how the parabolic curves closely approximate the position data, demonstrating the correctness of the horizontal dynamics model (7). Air-gap values here range between 10 and 27.5mm at 2.5mm intervals. In each case, the mover is started near \( x_3 = -50\text{mm} \) and accelerated to about \( x_3 = 50\text{mm} \) using a current of \( u_1 = -0.5A \). A comparison of the \( L_1(\bar{x}_1^i) \) estimates obtained using parabolic curve-fitting and the NPFSI estimator is found in Figure 7.

These results demonstrate that two different estimation techniques have predicted a similar response for the horizontal dynamics of the magnetic levitation system. This confirms the effectiveness of the NPFSI estimation technique and further indicates that the horizontal portion of the model describes the physical behavior of the system to a reasonable degree of accuracy. The estimates obtained for the \( \varepsilon = 0.05 \) case do exhibit increased divergence, but this would be expected since as the value of \( \varepsilon \) is increased, the estimator puts less emphasis on the estimation error and as a result, the convergence performance of the estimator
worsens.

**Verification of Vertical Dynamics**

In order to validate the vertical dynamics, we fix the mover at $x_3 = 0\text{mm}$ and only allow vertical motion. We also set $u_1 = 0$. As a result, only the vertical portion of (1) needs to be considered:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= G - L_4(x_1)u_2^2 - L_3(x_1)u_2 - L_2(x_1)
\end{align*}
\] (9)

From (9), we have that the current $\bar{u}_2$ needed to maintain the air-gap at a desired equilibrium $\bar{x}_1$ is found by solving the equation

\[
G - L_4(\bar{x}_1)u_2^2 - L_3(\bar{x}_1)u_2 - L_2(\bar{x}_1) = 0
\] (10)

or, what is the same,

\[
K_2(\bar{x}_1) + K_3(\bar{x}_1)u_2 + K_4(\bar{x}_1)u_2^2 - M_\ell G = 0.
\]

If the model (9) associated with the vertical dynamics is correct, the equilibrium current $\bar{u}_2$ predicted by (10),

\[
\bar{u}_2 = -K_3(\bar{x}_1) - \sqrt{K_3^2(\bar{x}_1) - 4K_4(\bar{x}_1)(K_2(\bar{x}_1) - M_\ell G)}
\]

\[
2K_4(\bar{x}_1)
\] (11)

should be close to the equilibrium current measured experimentally. The validation procedure for the vertical dynamics is now clear.

- The motion of the system is constrained (by hardware) to lie on the vertical axis with the mover positioned at $x_3 = 0\text{mm}$.
- Set $u_1 = 0$.
- For air-gaps in the set $\{\bar{x}_1^1, \ldots, \bar{x}_1^k\}$, the corresponding equilibrium currents $\{\bar{u}_2^1, \ldots, \bar{u}_2^k\}$ are measured.
• The NPFSI estimator is applied to (9) to estimate the constants $C_2$, $C_3$, and $C_4$.

• The estimates are used to determine the theoretical equilibrium currents by means of (11).

• The theoretical equilibrium currents are compared to the measured currents.

We choose the air-gap values to range between 10 and 25mm with 1mm increments. The NPFSI estimator is applied for three different values of $\varepsilon$. The estimation results are summarized in table 2. In Figure 8 the theoretical equilibrium currents found using the parameters $C_2, \ldots C_4$ estimated with $\varepsilon = 0.01$ are compared to the measured equilibrium currents. From the plot it is clear that while the model begins to diverge from the physical measurements for air-gaps smaller than 15mm, within the range between 15 and 25mm the model accurately describes the behavior of the magnetic levitation system. The results therefore validate the vertical dynamics of the model within the range of 15 and 25mm and provides further proof regarding the effectiveness of the NPFSI technique.

The divergence for smaller air-gaps is most likely the result of physical uncertainties that are not taken into account within the model. In the following section, evidence is provided that suggests that the cogging force accounts for the bulk of the uncertainty.

Analysis of the cogging force

We now seek to determine the source of the discrepancy, observed in Figure 8, between theoretical and measured equilibrium currents at air-gaps smaller than 15mm.

The cogging force of a linear synchronous motor is defined in [5] to be the force produced by the interaction between the teeth of the stator and the edges of the permanent magnets of the mover. It is a periodic function of the horizontal position of the mover over the slot pitch of the stator. Equation (12) provides a good mathematical representation of this cogging force $F_x^c$:

$$F_x^c = \xi(x_1) \sin \left( \frac{\pi}{t_1} x_3 \right)$$

(12)

The function $\xi(x_1)$, representing the peak magnitude of the cogging force, is typically inversely proportional to the air-gap $x_1$, meaning that the cogging force gets stronger as the
mover gets closer to the stator. Notice that the peaks of the cogging force occur at odd integer multiples of \( t_1/2 = 9.525 \text{mm} \). Hence, if the mover is held at a constant position \( x_3 = 28.575 \text{mm} = 3t_1/2 \), then the total normal force exerted by the PMLSM on the surface of the mover is given by

\[
F_n(x_1) \bigg|_{x_3=3t_1/2} = K_2(x_1) + K_3(x_1)u_2 + K_4(x_1)u_2^2 + \xi(x_1)
\]

where, as before, we are setting \( u_1 = 0 \). On the other hand, when \( x_3 = 0 \text{mm} \), the cogging force vanishes and thus the normal force should accurately be represented by our nominal model:

\[
F_n(x_1) \bigg|_{x_3=0} = K_2(x_1) + K_3(x_1)u_2 + K_4(x_1)u_2^2.
\]

Since the unknown constants \( C_2, C_3, \) and \( C_4 \) in the functions \( K_2, K_3, \) and \( K_4 \) have already been estimated in the previous section, to estimate \( \xi(x_1) \) we let \( \hat{K}_2(x_1) = K_2(x_1) + \xi(x_1) \), estimate \( \hat{K}_2 \) and get \( \xi(x_1) = \hat{K}_2(x_1) - K_2(x_1) \). This simple idea is the basis of the next procedure.

- The motion of the system is constrained (by hardware) to lie on the vertical axis with the mover positioned at \( x_3 = 28.575 \text{mm} = 3t_1/2 \).
- The constants \( C_3 \) and \( C_4 \) are assumed to be known and equal to the values estimated in the previous section. The constant \( C_2 \) is assumed to be unknown and is estimated by applying the NPFSI estimator to (9).
- The value of \( C_2 \) just found is used to generate an approximation of \( \hat{K}_2(x_1) \) as

\[
\hat{K}_2(x_1) = C_2 \tilde{\lambda}(x_1)B_{pmyl}(x_1)^2.
\]

- The difference \( \hat{K}_2(x_1) - K_2(x_1) \) represents a rough estimate of the peak of the cogging force as a function of the air-gap.

Applying the procedure above and using \( \varepsilon = 0.01 \) in the NPFSI estimator we get \( C_2 = 769.99 \). This gives the estimate of \( \xi(x_1) \) depicted in Figure 9. This clearly illustrates that the
estimated peak of the cogging force is appreciable (greater than 2N) when the air-gap is smaller than 15mm, and it is otherwise negligible for larger air-gaps.

This is confirmed in Figure 10, where the theoretical equilibrium currents are compared with the actual measurements of $u_2$ with and without the horizontal offset. While the two predictions are identical over most of the desired air-gap range, they diverge at air-gaps smaller than 15mm. Since the value of the normal force over the range of operation is of the order of 10N, it is clear from Figure 9 that, within the air-gap range between 15mm and 25mm, the cogging force at each horizontal position is a relatively small percentage of the total force, so it can be ignored in this range. However, Figure 9 indicates that for smaller air-gaps the discrepancy may become significant very quickly.

**Verification of Complete Model Dynamics**

With the horizontal and vertical dynamics of the magnetic levitation model verified separately, the final task is to confirm that all of the model parameters can be estimated simultaneously using the NPFSI estimator and still result in a valid 2-DOF model.

The reduced-order NPFSI estimator is therefore applied across the entire range of operation simultaneously at 3 values of $\epsilon$, in order to simultaneously estimate the 4 model parameters. The results are provided in table 3.

The complete 2-DOF model generated from the above parameter estimates is then verified using the horizontal and vertical techniques that were previously described. Figure 11 compares the estimate of $L_1(x_1)$ generated from the complete model with the estimates that were obtained at fixed air-gaps, while Figure 12 compares the theoretical equilibrium currents $u_2$ produced by the complete model with what was actually measured.

The comparisons show that the complete model generated using the full-span NPFSI estimator describes the physical behavior of the system to a reasonable degree of accuracy over the desired air-gap range. However, it should be noted that the resulting horizontal dynamics produced by the 2-DOF calibration procedure appear to diverge from what was estimated in the fixed air-gap case. This indicates that the decoupled estimates may be more accurate than the full-span case, or that the full-span case may have lacked sufficient persistency of excitation. As a result, elements of both the full parameter estimator and
the fixed air-gap estimator will serve as the 2-DOF calibration procedure in future control experiments that require accurate system modelling.

The effectiveness of the complete 2-DOF calibration procedure is illustrated in Figures 13 and 14, where the horizontal and vertical states measured from the actual system are compared over time with the predictions made by the final model (enhanced using estimated parameters). The reference signals used to generate these plots were the same used in generating the original persistency of excitation. Given that the plant is open-loop unstable, these results confirm once more the accuracy of our mathematical model.

**NONLINEAR CONTROLLER DESIGN AND IMPLEMENTATION**

In [3], two nonlinear stabilizers were presented, together with two procedures to rigorously estimate the range of operation of the associated closed-loop systems. Here we focus on the second controller presented in [3], which is based on feedback linearization, and implement a modification of it which addresses the presence of constant disturbances due to friction and bearing misalignment on the linear guides. Besides achieving set-point stabilization, we want the controller to also achieve sinusoidal tracking.

For convenience, we rewrite the model (3) incorporating two parameters $\Delta_1$ and $\Delta_2$ representing constant uncertainties due to friction and bearing misalignment.

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= G - C_4 \frac{\lambda(x_1) \coth^2(\frac{\pi(x_m + x_1))}{K_c(x_1)} [u_1^2 + u_2^2] + \Delta_2}{K_c(x_1)} \\
&\quad + C_3 \frac{\lambda(x_1) B_{pmy1}(x_1) \coth(\frac{\pi(h_m + x_1))}{u_2}}{K_c(x_1)} \\
&\quad - C_2 \lambda(x_1) B_{pmy1}(x_1)^2 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -C_1 \frac{\lambda(x_1)}{K_c(x_1) \sinh(\frac{\pi(h_m + x_1))} u_1 + \Delta_1
\end{align*}
\] (13)
Consider next the feedback transformation

\[
\begin{align*}
  u_1 &= -\frac{v_1}{L_1(x_1)} \\
  u_2 &= \frac{L_3(x_1) + \sqrt{R(x_1, v_1, v_2)}}{2L_4(x_1)}
\end{align*}
\]  

where

\[
R(x_1, v_1, v_2) = L_3(x_1)^2 + 4L_4(x_1)(-v_2 - L_4(x_1)U(x_1, v_1) - L_2(x_1) + G)
\]

\[
U(x_1, v_1) = \left(\frac{v_1}{L_1(x_1)}\right)^2.
\]

The resulting system reads as

\[
\begin{align*}
  \dot{x}_1 &= x_2 \\
  \dot{x}_2 &= v_2 + \Delta_2 \\
  \dot{x}_3 &= x_4 \\
  \dot{x}_4 &= v_1 + \Delta_1.
\end{align*}
\]  

with output \( y = (x_1, x_3) \). Before proceeding further with the design of \( v_1 \) and \( v_2 \), it is well to note that the feedback transformation (14) is well defined on the set \( \mathcal{A} = \{(x, v) : R(x_1, v_1, v_2) \geq 0, L_1(x_1) \neq 0, L_4(x_1) \neq 0\} \). Thus, in order for our controller to make sense, after choosing \( v_1 \) and \( v_2 \) we need to estimate what is the range of operation of the device, that is, the largest set of feasible initial conditions \( x(0) \) guaranteeing that \( (x(t), v(t)) \in \mathcal{A} \) for all positive time. This is precisely what was done in Procedure 2 in [3]. Owing to the fact that the choice of \( v \) here is different than that in [3], Procedure 2 in [3] needs to be modified. Doing this is beyond the scope of this paper (and will be done elsewhere), so we rather focus on showing experimentally that our controller is well-defined over the range of interest.

We next choose \( v_1 \) and \( v_2 \) to make \( x_1 \) and \( x_3 \) track a constant step or a sinusoid of fixed frequency \( \omega_0 \) (or a combination of the two) while rejecting the constant disturbances \( \Delta_1 \), \( \Delta_2 \). This control specification is best posed as a linear output regulation problem [6], [7], [8].
where the exosystem is

\[
\begin{align*}
\dot{w}_1 &= w_0 w_2 \\
\dot{w}_2 &= -w_0 w_1 \\
\dot{w}_3 &= w_0 w_4 \\
\dot{w}_4 &= -w_0 w_3 \\
\dot{w}_5 &= 0 \\
\dot{w}_6 &= 0 \\
\dot{w}_7 &= 0 \\
\dot{w}_8 &= 0 \\
\end{align*}
\]

(17)

and the output to be regulated is \( e = (e_1, e_2) = (x_1 - r_v, x_3 - r_h) \). The internal model is made of two copies of the same system:

\[
\begin{align*}
\dot{\xi}_v &= \phi \xi_v + N e_1, \quad y_2 = \Gamma \xi_v \\
\dot{\xi}_h &= \phi \xi_h + N e_2, \quad y_1 = \Gamma \xi_h \\
\end{align*}
\]

(18)

where

\[
\phi = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\omega_0^2 & 0 \end{bmatrix}, \quad N = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \Gamma = [1 \ 0 \ 0].
\]

It is useful to think of the \( \xi_v \) and \( \xi_h \) subsystems in (18) as being two internal models for the vertical and horizontal dynamics, respectively. For simplicity of implementation, we exploit the fact that \( w_1, \ldots, w_4 \) in the exosystem (17) and \( x_1, \ldots, x_4 \) in (16) can be assumed to be available for feedback and complete the output regulator design by letting

\[
\begin{align*}
v_1 &= K_h \begin{bmatrix} e_2 \\ \dot{\xi}_h \end{bmatrix}, \quad v_2 = K_v \begin{bmatrix} e_1 \\ \dot{\xi}_v \end{bmatrix},
\end{align*}
\]

(19)
where $K_h$ and $K_v$ are chosen so as to stabilize the augmented system

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= v_2 + \Gamma \xi_v \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= v_1 + \Gamma \xi_h \\
\dot{\xi}_v &= \phi \xi_v + N x_1 \\
\dot{\xi}_h &= \phi \xi_h + N x_3.
\end{align*}
\]

The final controller is thus given by (14), (18), and (19). Its block diagram is depicted in Figure 15.

With the design in place, the nonlinear controller can now be implemented and its performance evaluated.

**EXPERIMENTAL RESULTS**

The nonlinear controller was implemented using WINCON. Its ability to track step and sinusoidal responses was evaluated and compared with some linear counterparts.

**Set-Point Stabilization Performance of Nonlinear Controller**

Using the following values for the horizontal and vertical controller gains in (19),

\[
\begin{align*}
K_h &= \begin{bmatrix} -1182 & -55 & -154440 & -45128 & -11924 \end{bmatrix} \\
K_v &= \begin{bmatrix} -1662 & -65 & -360360 & -103350 & -20332 \end{bmatrix}
\end{align*}
\]

the resulting nonlinear controller was used to stabilize the system to a series of set-points over the ranges of $[-50\text{mm}, 50\text{mm}]$ horizontal and $[15\text{mm}, 25\text{mm}]$ vertical. Figure 16 shows the response of the vertical system states to the step commands in addition to the absolute vertical positioning accuracy. Figure 17 displays the same results for the horizontal system states. Finally, Figure 18 demonstrates the amount of current input required to achieve these results. The plots demonstrate the effectiveness of the nonlinear controller in performing set-point stabilization to the required 0.1mm accuracy with settling time below
3s over a wide operating range. Different experiments, not reported here, showed that the controller was capable of driving the tracking error to within encoder resolution (10µm) in about 10s.

Although no additional controller specifications were set, it should be noted that while the present controller gains produce short transient, the overshoot on the vertical steps is quite large and could potentially limit the range of motion. By making the following adjustment to the vertical controller gains,

\[ K_v = [-1592 \quad -81 \quad -55736 \quad -11573 \quad -15825] \]

Figure 19 shows how the step response now exhibits much smaller vertical overshoot at the cost of a longer settling time (still below 3s). Depending on the final industrial application, either the overshoot or settling time may be a more important factor. The results show that controller adjustments can be made to accommodate either requirement.

The superior performance of the nonlinear controller is clearly visible when compared with a linear counterpart. The linear controller under consideration for this comparison is the same linear output regulator from (19), but without the addition of the feedback transformation (14). In order to make a fair comparison, the model in (1) is linearized around a set-point \( \bar{x} \) and an output regulator with the same structure as (18)-(19) is designed so that the resulting closed-loop system poles coincide with the poles that the nonlinear controller induces in the feedback linearized plant. Next, various step responses are measured with steps of varying magnitude, ending at \( \bar{x} \). Figure 20 shows the results. The first step response is small, starting from the initial condition of \( (0.026m, 0, 0, 0) \) and ending at \( \bar{x} = (0.024m, 0, 0.005m, 0) \). The second step response is larger, starting at the same initial condition as before but terminating at \( \bar{x} = (0.018m, 0, 0.020m, 0) \).

The results show that while both the linear and nonlinear controller can handle a small step command, the linear response demonstrates a much larger transience and settling time. When subjected to the larger step command, the linear controller makes the plant unstable while the nonlinear controller still exhibits excellent performance. It should be noted that around 5.5 seconds, the instability exhibited by the linear controller causes a safety mecha-
nism within the real-time code to shut-down the system, which is why the oscillations halt at that point in time within Figure 20. An equivalent comparison was attempted between the nonlinear controller and the PID controller that was implemented to generate the necessary persistency of excitation for the system ID procedure. However, since the PID performed very poorly, it was clear that a detailed comparison was not necessary.

**Sinusoidal Tracking Performance of Nonlinear Controller**

For the tracking experiments, we choose $r_h(t)$ and $r_v(t)$ to be sinusoidal signals with amplitude 30mm and 5mm, respectively, and a frequency of $\omega_0 = 1.5\pi$ rad/sec. The vertical reference $r_v(t)$ also includes an offset of 20mm. The same controller gains from the set-point experiments were employed in the nonlinear controller. Figure 21 summarizes the tracking results obtained from the nonlinear controller. The horizontal tracking error averages about 0.11mm with occasional peaks that reach 0.4mm. The vertical tracking meanwhile averages 0.24mm with peaks that reach 0.8mm.

The nonlinear tracking performance was deemed satisfactory despite the fact that the vertical error is about twice the desired specification. While adjustments to the tracking performance were attempted, no significant improvements were achieved beyond the results that were presented. This is likely due to hardware limitations, namely the Coulumb friction of the linear guides and their imperfect alignment. The tracking performance of the nonlinear controller is significantly better than that of a basic PID arrangement that was utilized in generating the original persistency of excitation required for the system ID experiments. Figure 22 summarizes the tracking results obtained when the PID-based controller is used to track the same sinusoidal references as in the nonlinear case.

It can clearly be seen that the PID-based controller produces significantly more tracking error. Specifically, the average horizontal error is now about 0.19mm, while the vertical error now approaches 0.9mm. The nonlinear controller has therefore been shown to produce an improvement of at least 50% in the tracking error when compared to the PID arrangement.
CONCLUSIONS AND FUTURE WORK

A 2-DOF PMLSM-based magnetic levitation apparatus was presented as a first-generation test platform, in order to implement and verify the concepts that were laid out in [3].

The model verification results in this paper have shown that when a calibration procedure derived from the work presented in [4] is applied to the 2-DOF system, the state-space model from (1) can accurately predict the behavior of the physical system in an air-gap range between 15 and 25mm and a horizontal range between \(-50\)mm and 50mm. For air-gap values any smaller than the 15mm limit, the effects of uncertainties such as the cogging force become significant and require representation within the modeling.

With a sufficiently accurate model of the magnetic levitation system verified, a nonlinear controller was implemented based on feedback linearization and output regulation. The experimental results demonstrate that the nonlinear controller is capable of satisfactory set-point and sinusoidal tracking with a performance that exceeded some linear counterparts.

These results show promise for the remaining part of the research which entails implementing more sophisticated next-generation magnetic levitation hardware based on the PMLSM concept. At present, Quanser Inc. is nearing completion of a second-generation maglev apparatus with 3-DOF and constructed from 4 PMLSM’s. Once the 3-DOF apparatus is complete, the model verification and nonlinear control design procedure presented in this article will be implemented on this new test platform with more rigid control specifications.

Future generations of the maglev apparatus beyond the 3-DOF implementation will increase the number of degrees-of-freedom, decrease the dependence on linear guides and bearings, and produce a prototype more suitable for industrial applications.

References


Figure 1: 2-DOF magnetic levitation hardware implementation. The PMLSM stator and 3-phase coils are clearly visible at the top of the aluminum frame. The PMSLM mover is attached to a movable platform, which is visible near the center of the structure.
Figure 2: Close-up of horizontal linear guides. The linear guides secure the mover to the platform and allow for horizontal motion. The horizontal position sensor is also visible.
Figure 3: Close-up of vertical linear guides. The linear guides secure the platform and allow for vertical motion. One of the vertical stoppers is also visible in-between the guides.
Figure 4: Inertial frame of a single PMLSM. While the horizontal and vertical motion is along the $x$ and $y$-axis, motion is defined in the state-space model in terms of $d$ and $g$. 
Figure 5: Simplified block diagram of closed-loop system subject to PID control. The error between the vertical and horizontal positions of the platform relative to a reference are sent to PID controllers to produce currents $i_d$ and $i_q$. These currents are then converted into 3-phase current which is in turn sent to the physical apparatus. This setup is used to provide the required persistency of excitation for the parameter estimations.
Figure 6: Curve-fitting of parabolas to actual horizontal position data. The data points are clearly well approximated by the parabolas.
Figure 7: Comparison of various estimates of $L_1(x_1)$. Despite the increased divergence when $\varepsilon = 0.05$, the NPFSI estimator produces results that compare quite well with the method based on parabolic interpolation.
Figure 8: Comparing measured $i_d$ currents and theoretical predictions. The mathematical model accurately predicts the measured current over a wide air-gap range, and only begins to diverge for air-gaps smaller than 15mm.
Figure 9: Estimate of the peak cogging force over the entire air-gap range. Over most of the range the cogging force is below 1N and therefore negligible. As the air-gap is reduced to below 15mm, the effect becomes significant.
Figure 10: Measured and theoretical equilibrium $i_d$ currents at each air-gap. The divergence between the two offset positions is clearly visible.
Figure 11: Comparison between various estimates of $L_1(g)$. The 2-DOF calibration procedure, while still producing a reasonable prediction, seems to diverge from the previous results. This may be the result of coupling or insufficient persistency of excitation.
Theoretical Prediction of $I_d$ Using 2DOF Calibration Procedure

Experimental Measurement of $I_d$

Figure 12: Comparing predicted $i_d$ current with actual measurement. Note how the predicted currents still match the measured values just as accurately as before.
Figure 13: Comparison between the actual vertical position and speed states produced by exciting the apparatus and the predictions made by the final model.
Figure 14: Comparison between the actual horizontal position and speed states produced by exciting the apparatus and the predictions made by the final model.
Figure 15: Simplified block diagram of nonlinear controller. The tracking errors are fed to two internal models. Following the internal model principle, a state feedback controller is designed which stabilizes the cascade of plant and internal models when $\Delta_1 = \Delta_2 = r_v = r_h = 0$. The resulting control input $(v_1, v_2)$ is then used to generate $(u_1, u_2) = (i_q, i_d)$ by means of a feedback transformation. Finally, the pair $(i_q, i_d)$ is converted to 3-phase currents which are fed to the LCAM’s to control the maglev apparatus.
Figure 16: Plots of vertical system state responses to the set-point commands. The vertical positioning accuracy is also included. It is clear that set-point stabilization is achieved with the required 0.1mm accuracy. While the transient is very brief, the high overshoot could be unacceptable in some situations.
Figure 17: Plots of horizontal system state responses to the set-point commands. The horizontal positioning accuracy is also included. It is clear that set-point stabilization is achieved with the required 0.1mm accuracy. The transient and overshoot also appear to be acceptable.
Figure 18: Plots of current inputs required for set-point commands. The amount of peak and average current required falls well within the performance limits of the LCAM’s.
Figure 19: Plots of vertical step response when controller gains are adjusted. The overshoot is now much smaller at the cost of a longer settling time.
Figure 20: Plots comparing the step response performances of the linear and nonlinear controllers. It is clear that while both controllers can handle the small step response (though the linear controller does poorly), the linear controller yields instability for the larger step while the nonlinear controller is successful.
Figure 21: Plots showing tracking performance of nonlinear controller.
Figure 22: Plots showing tracking performance of PID-based controller. Note how the tracking error is now significantly worse than the nonlinear controller.
Table 1: Specifications for 2-DOF Magnetic Levitation Hardware

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<th>Parameter</th>
<th>Symbol</th>
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<td>Stator slot pitch</td>
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<td>Vertical Mover Mass</td>
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Table 2: VERTICAL MODEL PARAMETERS AT FIXED HORIZONTAL POSITION USING REDUCED-ORDER NPFSI ESTIMATOR

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Table 3: FINAL MODEL PARAMETERS ESTIMATED USING NPFSI

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