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A New Approach to the Design of Dynamic Output Feedback Stabilizers for LTI Systems

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***Abstract*— We present a new state-space approach for designing a dynamic output feedback control law which stabilizes a class of linear time invariant systems. All the states of the given system are not measurable and only the output is used to design the stabilizing control law. In the design scheme, however, we first assume that the given system can be stabilized by a feedback law composed of the output and its derivatives of a certain order. Beginning with this assumption, we systematically construct a dynamic system which removes the need of the derivatives. The actual order of the constructed dynamic feedback controller is the dimension of the output times the order of derivatives that are necessary. Therefore, it may be useful for order reduction of dynamic controllers. A set-point regulation problem for a magnetic levitation system is also solved without using the velocity measurement.**

I. Introduction

In this paper, we consider the stabilization problem of a system represented by

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

where x is the state in $\mathbb{R}^n$; u is the input in $\mathbb{R}^m$; y is the measurable output in $\mathbb{R}^p$.

We suppose that the system (1) is not able to be stabilized by any *static* output feedback. While the measurable states are not sufficient to design a stabilizing static control law, we assume that, if the output and 'its derivatives of a certain order' are available to be used, then a static feedback exists for stabilization. In this paper, we present a new way to replace the required derivatives by adding some dynamics in the feedback. This is, in fact, inspired by [1], where a passivity-based dynamic output feedback control has been proposed for inherently non-passive LTI systems by virtue of paralleling a feedforward compensator. In [1], it has also been observed that, when a system is stabilized by a proportional-derivative control, the derivative term can be replaced[1] with a compensator which has the same dimension as the system's input. The idea of replacing the derivative term is further exploited in this paper up to any order.

The only assumption in this paper is the following.

Assumption 1: Let us define

$$G_k := \begin{bmatrix} K_0 & K_1 & \cdots & K_k \end{bmatrix} \text{ and } H_k := \begin{bmatrix} C \\ CA \\ \vdots \\ CA^k \end{bmatrix}.$$

For the system (1), there exists an integer r $(1 \leq r)$ such that

$$A_r := A + BG_rH_r \quad \text{is Hurwitz.}$$

$\Diamond$

Remark 1: It is presumed in this assumption that $r \geq 1$ because, when Assumption 1 holds with $r = 0$, the system (1) can be trivially stabilized by a static output feedback without using additional dynamics. On the other hand, if the system (1) is stabilizable and observable, then Assumption 1 trivially holds with $r = n-1$. Indeed, in this case, H_r is left-invertible due to observability, and thus, there always exists G_r with which Assumption 1 holds. $\Diamond$

In the next section, a dynamic output feedback controller is presented for system (1) under Assumption 1, followed by a recursive algorithm to design the gains of the proposed controller in a systematic manner. Section III illustrates two design examples with simulation results. Conclusions are found in Section IV.

II. Main Results

For the system (1) satisfying Assumption 1, we propose a dynamic output feedback controller of order pr, which has the form of

$$\begin{aligned} \dot{\lambda} &= \Psi_a y + \Psi_b \lambda, \qquad \lambda \in \mathbb{R}^{pr}, \\ u &= \Phi_a y + \Phi_b \lambda. \end{aligned} \tag{2}$$

The output feedback stabilization problem is solved if we find $\Psi = [\Psi_a, \Psi_b]$ and $\Phi = [\Phi_a, \Phi_b]$ such that the following

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[1] The design of a dynamic system for replacing the velocity measurement has been studied by several authors [2–5].

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closed-loop system

$$\begin{aligned}\dot{x} &= Ax + B\Phi_a Cx + B\Phi_b\lambda \\ \dot{\lambda} &= \Psi_a Cx + \Psi_b\lambda\end{aligned} \tag{3}$$

is exponentially stable.

In the subsequent part of the paper, we propose a new way to design the matrices Ψ and Φ. Therefore, the main contribution of the paper is summarized as follows.

Theorem 1: For the system (1) satisfying Assumption 1, there exists a dynamic output feedback stabilizing controller (2) with additional λ-dynamics of order $(p \times r)$. ♢

The idea of constructing the controller (2) is to assume, temporarily in the beginning, that $H_r x$ is available for measurement. This makes the output feedback stabilization problem be solved by the static gain found in Assumption 1. Next, we change the temporary assumption such that $H_{r-1}x$ is available for measurement but $H_r x$ is not. (This implies that $CA^i x$, $i = 0, \cdots, r-1$, is measurable but $CA^r x$ is not.) Then, the control law designed at the previous step, where we assumed that $H_r x$ is measurable, is not implementable because it depends on the signal $CA^r x$. Hence, we separate the term $CA^r x$ from the control law and design additional dynamics with which the use of $CA^r x$ is eliminated. In the next step, we proceed by assuming that $H_{r-2}x$ is measurable but $CA^{r-1}x$ is not. This recursion goes to the end if we get a dynamic controller that requires only the true measurement of $H_0 x = Cx$ but not others.

The recursion begins by the following initial step.

A. Initial Step

When the $H_r x$ is measurable, we easily obtain the following stable closed loop system S_r with the gain G_r from Assumption 1:

$$S_r : \begin{cases} u &= G_r H_r x \\ &= G_{r-1}H_{r-1}x + K_r(CA^r x) \\ \dot{x} &= A_r x = (A + BG_r H_r)x \\ &= Ax + BG_{r-1}H_{r-1}x + BK_r(CA^r x). \end{cases} \tag{4}$$

Now, we assume that $H_{r-1}x$ is available for measurement but $CA^r x$ is not. Then, by introducing v, we decompose the system S_r into the term including $CA^r x$ and the rest (as follows):

$$u = G_{r-1}H_{r-1}x + K_r v \tag{5a}$$

$$\dot{x} = Ax + BG_{r-1}H_{r-1}x + BK_r v. \tag{5b}$$

If the following dynamic system is appended to (5b)

$$\dot{\lambda} = -CA^{r-1}BG_{r-1}H_{r-1}x - (I + CA^{r-1}BK_r)v \tag{6a}$$

$$\bar{y} = CA^{r-1}x + \lambda, \tag{6b}$$

then the augmented system (5b)–(6a) is stabilized by $v = D_r\bar{y}$ where D_r is chosen so that the following matrix is Hurwitz

$$\begin{bmatrix} A_r & -A_r BK_r \\ CA^r & -CA^r BK_r - D_r \end{bmatrix}. \tag{7}$$

Proof of Initial Step.
First of all, note that

$$\begin{aligned}\dot{\bar{y}} &= CA^{r-1}\dot{x} + \dot{\lambda} \\ &= CA^{r-1}(Ax + BG_{r-1}H_{r-1}x + BK_r v) \\ &\quad - (CA^{r-1}BG_{r-1}H_{r-1}x + CA^{r-1}BK_r v + v) \\ &= CA^r x - v.\end{aligned}$$

We now define

$$\xi := x + BK_r\bar{y} \tag{8}$$

and change coordinates $[x^T \ \ \lambda^T]^T$ into $[\xi^T \ \ \bar{y}^T]^T$. Then

$$\begin{aligned}\dot{\xi} &= A_r\xi - A_r BK_r\bar{y} \\ \dot{\bar{y}} &= CA^r\xi - CA^r BK_r\bar{y} - v.\end{aligned} \tag{9}$$

Since the matrix A_r is Hurwitz, the system (9) can be stabilized by $v = D_r\bar{y}$ with an appropriate gain D_r making the matrix (7) Hurwitz. For example, $D_r = d_r I$ with sufficiently large $d_r > 0$ always performs this task. ♢

Consequently, we obtain the closed loop system S_{r-1} as follows:

$$\begin{cases} u = G_{r-1}H_{r-1}x + K_r D_r(CA^{r-1}x + \lambda) \\ \quad = \big(G_{r-1} + [\, 0_{m\times p(r-1)} \ \ K_r D_r]\big)H_{r-1}x + K_r D_r\lambda \\ \dot{x} = Ax + B\big(G_{r-1} + [\, 0_{m\times p(r-1)} \ \ K_r D_r]\big)\, H_{r-1}x \\ \qquad + BK_r D_r\lambda \\ \dot{\lambda} = -\big(CA^{r-1}BG_{r-1} + [\, 0_{p\times p(r-1)} \ \ M_\lambda\,]\big)\, H_{r-1}x \\ \qquad - M_\lambda\lambda \end{cases} \tag{10}$$

where $M_\lambda = (I + CA^{r-1}BK_r)D_r$. The above system (10) is stable because its system matrix is similar to the matrix (7).

B. Recursive Design of Output Feedback Controller

We assume that, with some integer k between 1 and r, it holds that $H_k x$ is measurable and the following output feedback controller of order $p(r-k)$ stabilizes system (1) exponentially:

$$\begin{aligned}\dot{\lambda} &= \Psi_{k,a}H_k x + \Psi_{k,b}\lambda, \\ u &= \Phi_{k,a}H_k x + \Phi_{k,b}\lambda,\end{aligned} \tag{11}$$

where $\Phi_{k,a}$, $\Phi_{k,b}$, $\Psi_{k,a}$ and $\Psi_{k,b}$ are matrices of appropriate dimension. In other words, the closed-loop system

$$S_k : \begin{cases} \dot{x} &= Ax + B\Phi_{k,a}H_k x + B\Phi_{k,b}\lambda \\ \dot{\lambda} &= \Psi_{k,a}H_k x + \Psi_{k,b}\lambda \end{cases} \tag{12}$$

is exponentially stable, which can be concisely represented by

$$\dot{z} = A_k z, \tag{13}$$

where $z := [x^T, \lambda^T]^T$ and the Hurwitz matrix A_k is defined as

$$A_k = \begin{bmatrix} A + B\Phi_{k,a}H_k & B\Phi_{k,b} \\ \Psi_{k,a}H_k & \Psi_{k,b} \end{bmatrix}. \tag{14}$$

Now we postulate a new assumption that $H_{k-1}x$ is measurable but CA^kx is not, so that the controller (11) cannot be implemented. Thus, we separate the term CA^kx from the controller equation (11) and replace it by a new signal v to be designed as follows:

$$\begin{aligned}\dot{\lambda} &= \Psi_{k,a1}H_{k-1}x + \Psi_{k,b}\lambda + \Psi_{k,a2}CA^kx \\ &= \Psi_{k,a1}H_{k-1}x + \Psi_{k,b}\lambda + \Psi_{k,a2}v \\ u &= \Phi_{k,a1}H_{k-1}x + \Phi_{k,b}\lambda + \Phi_{k,a2}CA^kx \\ &= \Phi_{k,a1}H_{k-1}x + \Phi_{k,b}\lambda + \Phi_{k,a2}v,\end{aligned} \tag{15}$$

where $\Psi_{k,a} = [\Psi_{k,a1}, \Psi_{k,a2}]$ and $\Phi_{k,a} = [\Phi_{k,a1}, \Phi_{k,a2}]$. Then, the closed-loop system is rewritten by

$$\begin{aligned}\dot{x} &= Ax + B\Phi_{k,a1}H_{k-1}x + B\Phi_{k,b}\lambda + B\Phi_{k,a2}v \\ \dot{\lambda} &= \Psi_{k,a1}H_{k-1}x + \Psi_{k,b}\lambda + \Psi_{k,a2}v,\end{aligned} \tag{16}$$

or

$$\dot{z} = Fz + Lv \tag{17}$$

where

$$F = \begin{bmatrix} A + B\Phi_{k,a1}H_{k-1} & B\Phi_{k,b} \\ \Psi_{k,a1}H_{k-1} & \Psi_{k,b}\end{bmatrix},\ L = \begin{bmatrix} B\Phi_{k,a2} \\ \Psi_{k,a2}\end{bmatrix},$$

which is equivalent to (12) (or to (13)) if $v = CA^kx$. Note that $A_k = F + L[CA^k, 0_{p\times p(r-k)}]$.

The following theorem provides a key to the recursion in the sense that it shows how to replace CA^kx term by an additional dynamics.

Theorem 2: Suppose that system (16) (or, (17)) is exponentially stable if $v = CA^kx$; that is, the matrix A_k is Hurwitz. If the following dynamic system is appended to (16) (or, (17))

$$\begin{aligned}\dot{\eta} = &-CA^{k-1}B\Phi_{k,a1}H_{k-1}x - CA^{k-1}B\Phi_{k,b}\lambda \\ &- (I + CA^{k-1}B\Phi_{k,a2})v, \qquad \eta \in \mathbb{R}^p,\end{aligned} \tag{18a}$$

$$\bar{y} = CA^{k-1}x + \eta, \tag{18b}$$

then the augmented system (16), (18) (or, (17),(18)) is exponentially stabilized by

$$v = D_k\bar{y}, \tag{19}$$

where the matrix D_k is chosen such that

$$\begin{bmatrix} A_k & -A_kL \\ [CA^k \quad 0_{p\times p(r-k)}] & -CA^kB\Phi_{k,a2} - D_k\end{bmatrix} \tag{20}$$

is Hurwitz. ◊

Remark 2: Note that the matrix (20) always can be made Hurwitz by appropriate matrix D_k, which can be found by LMI tool or by choosing sufficiently large constant $d_k > 0$ and letting $D_k = d_kI$. ◊

Proof: With the control law (18) and (19), the closed-loop system is given by (17) and (18a) with (19). In order to analyze its stability, the closed-loop system is represented in the $(z, \bar{y})$-coordinates instead of (z, η). That is, the closed-loop system is now given by (17) and

$$\begin{aligned}\dot{\bar{y}} &= CA^{k-1}(Ax + B\Phi_{k,a1}H_{k-1}x + B\Phi_{k,b}\lambda + B\Phi_{k,a2}v) \\ &\quad - CA^{k-1}B\Phi_{k,a1}H_{k-1}x - CA^{k-1}B\Phi_{k,b}\lambda \\ &\quad - (I + CA^{k-1}B\Phi_{k,a2})v \\ &= CA^kx - v.\end{aligned}$$

Now we change the coordinates $(z, \bar{y})$ into $(\xi, \bar{y})$ once again with a new variable $\xi := z + L\bar{y}$. That is,

$$\begin{aligned}\dot{\xi} &= (Fz + Lv) + (LCA^kx - Lv) = A_kz \\ &= A_k\xi - A_kL\bar{y} \\ \dot{\bar{y}} &= [CA^k, 0_{p\times p(r-k)}]z - v \\ &= [CA^k, 0_{p\times p(r-k)}]\xi - [CA^k, 0_{p\times p(r-k)}]L\bar{y} - v \\ &= [CA^k, 0_{p\times p(r-k)}]\xi - CA^kB\Phi_{k,a2}\bar{y} - v \\ v &= D_k\bar{y}.\end{aligned}$$

Therefore, it is seen that if D_k is chosen such that the matrix (20) is Hurwitz, the above closed-loop system is exponentially stable. ■

Remark 3: As a result of Theorem 2, it follows that the overall closed-loop system, which is obtained from (16), (18) and (19), is exponentially stable. The single equation (21) is the closed-loop system, whose system matrix will become the matrix A_{k-1} in the next iteration step. ◊

The recursion procedure is now quite obvious. Since $k = r$ at the initial step, $\Psi_{r,a}$, $\Psi_{r,b}$ and $\Phi_{r,b}$ are null matrices (i.e., empty) and the controller (18) becomes just a static feedback $u = G_rH_rx$ (i.e., $\Phi_{r,a} = G_r$) from Assumption 1. Therefore, we have the Hurwitz matrix $A_r = A + BG_rH_r$. By Theorem 2, unmeasurable term CA^rx is replaced by the dynamic controller (18) and (19). Now, we regard the state η of (18) as the state λ of (11) (i.e. (6) and (10)) for the next iteration. (The next step begins with the equation (11).) In particular, from (10) it is obtained that

$$\begin{aligned}\Psi_{r-1,a} &= -\big(CA^{r-1}B\Phi_{r,a1} \\ &\qquad + [\ 0_{p\times p(r-1)},\ \ (I + CA^{r-1}B\Phi_{r,a2})D_r\ ]\big) \\ \Psi_{r-1,b} &= -(I + CA^{r-1}B\Phi_{r,a2})D_r \\ \Phi_{r-1,a} &= \Phi_{r,a1} + [\ 0_{m\times p(r-1)},\ \ \Phi_{r,a2}D_r] \\ \Phi_{r-1,b} &= \Phi_{r,a2}D_r\end{aligned}$$

where $\Phi_{r,a1} = G_{r-1}$ and $\Phi_{r,a2} = K_r$. Likewise, the iteration proceeds until we have a controller of (11) with $k = 0$. Therefore, we obtain the gains of (2) as follows:

$$\Psi_a = \Psi_{0,a}, \quad \Psi_b = \Psi_{0,b}, \quad \Phi_a = \Phi_{0,a}, \quad \Phi_b = \Phi_{0,b}.$$

$$\begin{bmatrix}\dot{x}\\ \dot{\lambda}\\ \dot{\eta}\end{bmatrix}=\begin{bmatrix} A+B\Phi_{k,a1}H_{k-1}+B\Phi_{k,a2}D_kCA^{k-1} & B\Phi_{k,b} & B\Phi_{k,a2}D_k\\ \Psi_{k,a1}H_{k-1}+\Psi_{k,a2}D_kCA^{k-1} & \Psi_{k,b} & \Psi_{k,a2}D_k\\ -CA^{k-1}B\Phi_{k,a1}H_{k-1}-(I+CA^{k-1}B\Phi_{k,a2})D_kCA^{k-1} & -CA^{k-1}B\Phi_{k,b} & -(I+CA^{k-1}B\Phi_{k,a2})D_k\end{bmatrix}\begin{bmatrix}x\\ \lambda\\ \eta\end{bmatrix}. \quad (21)$$

For convenience, we include a formula for the iteration:

$$\Psi_{k-1,a}=\begin{bmatrix}\Psi_{k,a1}+[\,0_{p(r-k)\times p(k-1)},\Psi_{k,a2}D_k\,]\\ \begin{pmatrix}-CA^{k-1}B\Phi_{k,a1}-[\,0_{p\times p(k-1)},\\ (I+CA^{k-1}B\Phi_{k,a2})D_k\,]\end{pmatrix}\end{bmatrix}$$

$$\Psi_{k-1,b}=\begin{bmatrix}\Psi_{k,b} & \Psi_{k,a2}D_k\\ -CA^{k-1}B\Phi_{k,b} & -(I_{p\times p}+CA^{k-1}B\Phi_{k,a2})D_k\end{bmatrix}$$

$$\Phi_{k-1,a}=\Phi_{k,a1}+[0_{m\times p(k-1)},\Phi_{k,a2}D_k]$$

$$\Phi_{k-1,b}=\begin{bmatrix}\Phi_{k,b}, & \Phi_{k,a2}D_k\end{bmatrix}.$$

III. Illustrative Examples

Example 1. We illustrate the proposed design method with a simple numerical example:

$$\begin{aligned}\dot{x}&=\begin{bmatrix}0&a&0\\0&0&1\\0&0&0\end{bmatrix}x+\begin{bmatrix}0\\0\\1\end{bmatrix}u\\ y&=\begin{bmatrix}1&0&0\end{bmatrix}x\end{aligned} \quad (22)$$

where $a=1$. The system (22) satisfies Assumption 1 with $r=2$. In fact, with the following control law

$$u=G_2H_2x=\begin{bmatrix}-50 & -40 & -11\end{bmatrix}H_2x, \quad (23)$$

the eigenvalues of the matrix $A_2=A+BG_2H_2$ are given by $\{-5,-3\pm j\}$. Hence, the closed loop system (22)–(23) is stable and we obtain $G_1=\begin{bmatrix}-50 & -40\end{bmatrix}$ and $K_2=-11$ for the iteration.

Now, in order to replace the CA^2x-term in H_2x as the initial step, we consider the matrix of (7) for the system (22). Indeed, with $D_2=30$, the matrix (7) is given by

$$\begin{bmatrix}0&1&0&0\\0&0&1&11\\-50&-40&-11&-121\\0&0&1&-19\end{bmatrix} \quad (24)$$

which is Hurwitz.

However, since the CAx-term in H_1x is neither measurable, we proceed one step further by Theorem 2. From the previous step, the parameters of (11) can be regarded as

$$\begin{aligned}\Psi_{1,a}&=[0\ \ -30],\ \Psi_{1,b}=-30,\\ \Phi_{1,a}&=[-50\ \ -370],\ \Phi_{1,b}=-330.\end{aligned} \quad (25)$$

With these parameters the matrix A_1 in (14) is given by

$$A_1=\begin{bmatrix}0&1&0&0\\0&0&1&11\\-50&-370&0&-330\\0&-30&0&-30\end{bmatrix}. \quad (26)$$

Hence, the gain D_1 is chosen such that the matrix in (20) is Hurwitz, which is achieved by $D_1=30$.

Therefore, with the following additional dynamics

$$\begin{cases}\dot{\lambda}=-900y-30\lambda-900\eta\\ \dot{\eta}=-30y-30\eta,\end{cases} \quad (27)$$

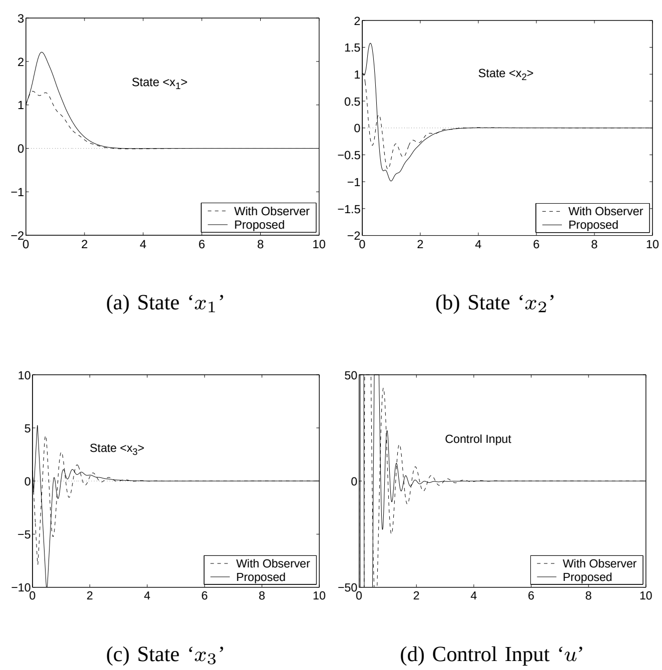
the stabilizing control law for (22) is obtained by

$$u=-11150y-330\lambda-11100\eta. \quad (28)$$

Figure 1 shows the simulation result (solid curve) of the proposed controller. The simulation is performed for $a=2$, although its nominal value is 1, to see some robustness property that the proposed controller might have. In the figure, we added a saturation whose level is 50, and also compared the plots with the results (dotted curve) obtained from the classical observer-based control (the Luenberger observer plus a state feedback) [6]. All the initial conditions of the systems are set to 1 while all the initial states of the additional dynamics and the observer are set to 0.

Example 2. Consider the linearized model of a magnetic ball levitation system [7]:

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=2800x_1-19600x_3\\ \dot{x}_3=-26.667x_3+2.4242u\\ \dot{x}_4=x_1-x_c\end{cases},\quad y_m=\begin{bmatrix}x_1\\x_3\\x_4\end{bmatrix} \quad (29)$$



(a) State ‘x_1’ (b) State ‘x_2’

(c) State ‘x_3’ (d) Control Input ‘u’

Fig. 1. Simulation Results (proposed: solid).

where x_1 is the distance of the ball from the electromagnet face; x_2 is the velocity of the ball; x_3 is the current in the magnet coil; x_4 is the integral of the position error $e = x_1 - x_c$ ($x_c = 7$), and u is the input voltage applied to the coil. The model (29) is obtained by linearizing the nonlinear dynamics about the point $x_1 = 7$, $x_2 = 0$, $x_3 = 1$. In addition, we assume that the states x_1, x_3 and x_4 are measurable.

To translate the equilibrium into the origin, we first define a new variable $z = \begin{bmatrix} x_1 - 7, & x_2, & x_3 - 1, & x_4 \end{bmatrix}^T$. Then, we can find an LQR controller of the form

$$u_z = -kz = -\begin{bmatrix} k_1 & k_2 & k_3 & k_4 \end{bmatrix} z \tag{30}$$

that stabilizes the system and minimizes the performance index $J = \int_0^\infty (z^T Q z + u_z^T \bar{R} u_z) dt$ [8]. For example, with $Q = \mathrm{diag}[1 \;\; 0 \;\; 1 \;\; 1]$ and $\bar{R} = 10$, the state feedback gain k is obtained as

$$\begin{aligned} k &= \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \end{bmatrix} \\ &= \begin{bmatrix} -9.464 & -0.179 & 43.862 & -0.316 \end{bmatrix}. \end{aligned} \tag{31}$$

With the control $u = u_z + 11$, the poles of closed-loop system are located at $\{-55.74, -49.38, -27.68, -0.20\}$.

Now, we will show that the system (29) can be stabilized by the proposed method without the measurement of z_2 that was used in (30). Note that if we design the additional system (2) with the output y_m, the constructed dynamics become a system of order 3. Since z_1 is the only state that should be differentiated, however, we let $y = z_1$ and $u_z = u_1 + u_2$ where

$$u_1 = -k_1 z_1 - k_2 z_2, \;\; u_2 = -k_3 z_3 - k_4 z_4. \tag{32}$$

By doing this we will show that one dimensional additional system is enough to stabilize the given system.

With the control input u_2 only, we first get the system matrices

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 2800 & 0 & -19600 & 0 \\ 0 & 0 & -133.0 & 0.767 \\ 1 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 2.424 \\ 0 \end{bmatrix}, \tag{33}$$
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}.$$

Next, we can write control law u_1 as

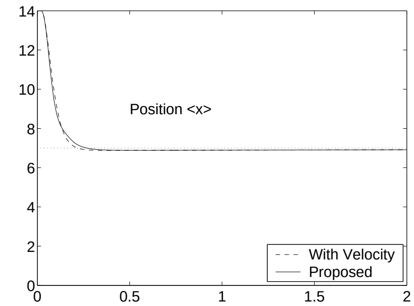
$$u_1 = -k_1 C z - k_2 C A z, \tag{34}$$

from which we can get G_1 that is necessary for the design of the compensator.

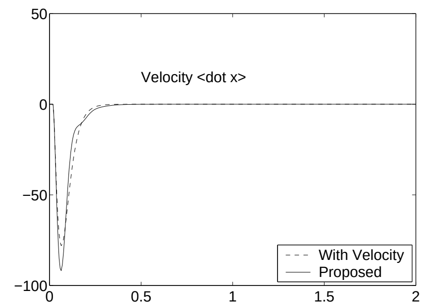
From the above equations we can obtain $K_0 = -k_1$ and $K_1 = -k_2$, and hence the system (6) is given by

$$\begin{cases} \dot{\lambda} = -v \\ \bar{y} = z_1 + \lambda \end{cases} \tag{35}$$

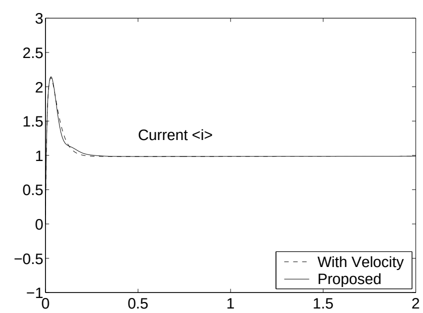
With $v = \psi \bar{y}$ for sufficiently large ψ, it can be seen that the system (33)–(35) is stabilized by $u_1 = -k_1 z_1 - k_2 \psi \bar{y}$ ($\psi \gg 1$). The poles of the closed-loop system for $\psi = 200$ are given by $\{-258.13, -28.856 \pm j51.467, -16.955, -0.1972\}$. As a result, we can conclude



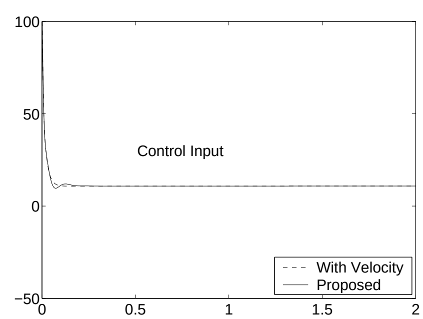
(a) State 'x_1'



(b) State 'x_2'



(c) State 'x_3'



(d) Control Input 'u'

Fig. 2. Simulation Results (proposed: solid).

that the system (29) can be stabilized by the following control input:

$$u = -k_1(x_1 - x_c) - k_3(x_3 - 1) - k_4 x_4 - k_2 \psi \bar{y} + 11.$$

Figure 2 shows the simulation results with the initial condition $x(0) = [14, 0, 0, 0]$ and $x_c = 7$. The performance of the proposed controller is compared with the controller (30) that requires the velocity information. In this figure, solid and dotted curves represent the result of the proposed controller and that of the controller (30), respectively.

IV. Conclusion

In this paper, we have presented a new recursive algorithm to design a dynamic output feedback control law which stabilizes linear time-invariant systems that can be stabilized by a static feedback of the output and its derivatives. Examples with simulation results are presented. From the proposed recursion algorithm, it is not difficult to develop an automated design package on a PC.

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References

[1] Y.I. Son, H. Shim, K. Park, and J.H. Seo. Passification of non-square linear systems using an input-dimensional dynamic feedforward compensator. *IEICE Trans. on Fundamentals*, E85-A(2):422-431, 2002.

[2] H. Kaufman, I. Bar-Kana, and K. Sobel. *Direct Adaptive Control Algorithms*. Springer-Verlag, 2nd edition, 1998.

[3] R. Kelly, R. Ortega, A. Ailon, and A. Loria. Global regulation of flexible joint robots using approximate differentiation. *IEEE Trans. Automat. Contr.*, 39:1222–1224, 1994.

[4] Y. Fujisaki, M. Ikeda, and K. Miki. Robust stabilization of large space structures via displacement feedback. *IEEE Trans. Automat. Contr.*, 46(12):1993–1996, 2001.

[5] H. Wong, M.S. de Queiroz, and V. Kapila. Adaptive tracking control using synthesized velocity from attitude measurements. *Automatica*, 37:947–953, 2001.

[6] C. T. Chen. *Linear System Theory and Design*. New York: Oxford Univerisity Press, 1984.

[7] Quanser Inc. MAGLEV experiment guide (online manual available at `http://www.quanser.com`).

[8] B.D.O. Anderson and J.B. Moore. *Optimal Control*. Prentice-Hall, 1989.